MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XXXIV. *January* 9, 1874. No. 3.

PROFESSOR CAYLEY, F.R.S., President, in the Chair.

Charles Becker, Esq., 112 St. Martin's Lane;
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Ralph Copeland, Ph.D., Parsonstown Observatory, Ireland;
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Major W. Palmer, R.E., Southborough, Tunbridge Wells;
Dr. J. C. Robertson, Monaghan, Ireland;
Capt. F. C. B. Robinson, R.N., Junior United Service Club; and
Rev. Thos. Thistlethwaite Smith, Thruxton Rectory, Hereford,

were balloted for and duly elected Fellows of the Society.

Dr. Edouard Heis, Münster;
Dr. Theodor Oppolzer, Vienna;
Dr. Johann Freidrich Julius Schmidt, Athens Observatory;
and Mons. C. Wolf, Paris Observatory,

were also balloted for and duly elected Associates of the Society.

On a proposed New Method of Treating the Lunar Theory. By Sir George Biddell Airy, K.C.B., Astronomer Royal.

It appears desirable to introduce the explanation of the method now proposed by a rapid survey of the methods hitherto employed.

In the whole range of physical mathematics, there is perhaps nothing more remarkable than the beauty of the geometrical

integrations, in the III. Book of *Newton's Principia*, for the Lunar Inequalities in Latitude and for the Lunar Variation; and the general accuracy of the results. It is clear also, from a few remarks in the 11th Section of the I. Book, and from an unexplained remark on the comparison of the inequalities of the satellites of other planets with those of our Moon in the III. Book, that Newton perfectly understood the origin of what are now called the terms of the second order, by which the Velocity of Progression of the Apse, and the Evection, are so much increased. But Newton published no numerical calculation of those quantities; and the theory was, so far, left imperfect. A more powerful calculus was necessary.

The want was supplied by the Differential Calculus, in the shape in which it was established among Continental mathematicians; and the particular form in which it was applied by Clairaut to the Lunar Theory exhibited at once the power of the Calculus and the ease of applying it. The simple form of Clairaut's differential equations for parallax and latitude opened out the entire process of extending the theory to any degree of accuracy, and showed at the same time the steps by which periodical inequalities of one form are deduced from the combination of periodical inequalities of other forms. I think that scarcely sufficient honour has been given to Clairaut for the formation of this special equation, without which the progress of the theory would probably have been very slow. Even now it is the best form in which a beginner can enter upon the studies of the Lunar Theory. Clairaut's theory gives the time in terms of the arc of longitude described, which is not without advantage in the treatment of equations of long period; but it requires a final reversion of series, in order to give the longitude in terms of the time. Mathematicians in the later part of the present century have preferred a form in which the Moon's ordinates are expressed immediately in terms of the time. I give my adhesion to this method; but at the same time I am anxious to offer my testimony to the value of the process so successfully introduced at a most critical point in the progress of the science.

The next important extensions of the theory were those of Laplace and Damoiseau: both founded on Clairaut's equation; both exhibiting the subordinate equations derived from the comparison of coefficients which are expressed by unexpanded algebraical fractions whose denominators are very complicated (the piles of these fractions, especially in Damoiseau's work, are appalling); both giving the first results in numerical values for the coefficients of numerous arguments which are multiples of longitude; both leaving in great obscurity the process by which the numerical solutions of these algebraical comparisons were obtained; and both giving the final results in terms depending on the time. Damoiseau, however, added to this investigation a work which demands our gratitude: a system of Lunar Tables expressly founded on the aggregation of simple periodical terms

having for arguments different multiples of the time. It was by use of these Tables (with small additions derived principally from Plana) that I conducted the great Reduction of Lunar Observations from 1750 to 1853, and deduced from them the corrections of the principal coefficients. Damoiseau's angular values were all expressed in the centesimal division of the quadrant: a method which possesses so many advantages that I hope for its adoption in future tables.

Plana's work, which followed, was not entirely pure in its method. It commences, for instance, with an application of theorems for the "variation of constants," here introduced with great advantage. But in the more advanced parts it may be described as established on the use of the time as the independent variable, and as exhibiting every coefficient in a series of algebraical terms without denominators. Viewed as leading to an algebraical result, this work was a great advance beyond all which had preceded it; and in numerical accuracy it is probable that something was gained.

I do not advert to the extensive investigations of Lubbock, because they were principally in the nature of verifications, adopting generally M. Plana's system. Nor do I consider the important questions raised by Professor Adams, because they are, in fact, a re-examination of specific points in a received theory. Professor Hansen's theory and tables require mention, principally in explanation of my reasons for almost omitting them from a view of the progress of the science. I attach the highest value to Professor Hansen's discovery of two inequalities in longitude produced by *Venus*; of which one is universally accepted, and the other, though controverted, still appears plausible. And I value the new equation which he introduced in the Moon's latitude. I believe also that the object which Professor Hansen originally proposed to himself, namely, the more rapid convergence of terms, has been (in some measure at least) attained. Yet I think that the general form of his theory, differing so much from the two systems which had preceded it, and presenting little facility for correcting elements from observations, is so far objectionable that it is not likely to be adopted by future lunar theorists; and that its introduction was, in fact, a retrograde step. But, in common with all who are practically concerned with lunar observations, I am grateful for his Lunar Tables, which, embodying the results of his own theory and the Greenwich corrections of elements, and published at a time when the existing tables were running wild, have been most beneficial to practical science.

But there remains one glorious work, almost superhuman in its labour, and perfect beyond others in the detailed exhibition of its results; the Lunar Theory of Delaunay. In this the time is adopted as the independent variable. The masses of undeveloped fractions here exhibited are greater than those of Damoiseau; the development in terms without denominators is more extensive

than that of Plana; and the numerical evaluation of every term is more complete than that of any preceding writer. Some terms to which we should have attached great interest are lost (at least for the present) by the untimely death of M. Delaunay.

Now, in all these works, so far as I have remarked, the following characteristics hold:—

(1). Each investigator has begun his work *de novo*, without making any use of the results of preceding investigators, even with the application of contingent corrections.

(2). Each investigator has used the fractions, in symbolical terms, to which I have alluded; and, by adherence to the symbolical form, has been compelled to expand them in series with rapidly increasing coefficients.

(3). The nature of the steps has compelled the investigators to decide the succession of their terms, not by numerical magnitude, but by algebraical order. And this has produced great inequality of convergence. Delaunay's smaller coefficients are probably correct, as he has exhibited them, to $0''{\cdot}0001$; but his larger terms converge so slowly that he has been compelled to supplement them by an assumed law of decrease; and they may perhaps be in error by almost $1''{\cdot}0000$.

(4). The mental labour in these operations is fearfully great. M. Plana once remarked to me, "Quelquefois, Monsieur, ces calculs me font presque perdre la tête."

(5). This labour cannot be alleviated, even in the examination of work done, by an amanuensis or assistant.

In consideration of these circumstances (which I have known, as well from examination of the works of others, as from my private investigations), I have long held the opinion that a Lunar Theory, in which every coefficient is expressed, from the very beginning of the process and throughout, by simple numbers, is very desirable. My ideas on this subject have by degrees assumed an orderly form; and I am now able to exhibit their leading points, as follow:—

(1*). I propose to assume Delaunay's final numerical expressions, for longitude, latitude, and parallax, with the addition of secular equations, as my fundamental numbers. These will be converted into other numerical expressions referred to more convenient units. To every number, as far as I think necessary, will be attached a symbolic term for contingent correction; in some cases considered as varying with the time. In all cases I assume that this correction will be so small that its first power will be sufficient. The secular terms will probably introduce cosines with sines of the same argument.

(2*). I propose to substitute these numbers with symbolical corrections in the equations in which the time is adopted as independent variable. The fractions to

which I have alluded will still occur, but not in a troublesome symbolical form. The greatest complication of denominators will be that of "a number with small symbolical correction attached to it;" which will be instantly converted into two terms without denominator. There will never be an infinite series.

(3*). The order of terms will be numerical; and, as far as I perceive, they will be equally accurate throughout.

(4*). The details of work will be very easy.

(5*). A great part of the work can be intrusted to a mere computer; and probably the whole can be examined, or can be repeated in duplicate, by such assistant.

To these I add,

(6*). I have strong confidence that equations of very long period may thus be examined with great severity, especially when there is reason to suspect that the form of the principal arguments may be slightly changed.

(7*). The result of the comparison of the terms in the mechanical or gravitational equations will be, a great number of equations for determining the numerical values of a great number of small quantities. I anticipate no difficulty in the solution; it is usually sufficient, for the determination of any one of the small quantities, to change (where necessary) the sign of its coefficient, so as to have all its coefficients with the same sign (the sign of the constant term being also changed), and to add all; neglecting all the other unknown quantities. In some cases, however, it may be necessary to treat two of these corrections in combination.

Though very late, I have actually begun a Lunar Theory in the shape which I have described. It is sufficiently possible that I may not be able to complete it; but I desire to leave it in such a state that a successor may be able to take it up successfully. For this purpose, I will enter into some further details as to the steps which I have made.

I. I refer all ordinates to an invariable plane and an invariable line in that plane; for instance, the plane of the ecliptic and the equinoxial line for the beginning of the year 1900.

II. I represent the masses of the Sun, the Earth, and the Moon, by the Greek letters σ, ε, μ. It is supposed that they are estimated, as is usual, by the acceleration which their attraction at distance 1 would produce in time 1. The letter σ' is accented to show that it is necessary to apply a term of contingent correction to the assumed Sun's mass, and similarly for ε' and μ'.

III. It is easily demonstrated that, to a very high degree of accuracy, the motion of the centre of gravity of Earth and Moon is subject to the same laws as the motion of a planet in that place, and that their action on Sun and planets is the same as if their mass were collected in that place.

IV. Explaining at present only a small part of the adopted

notation: A′, R′, V′, are the true major axis of orbit, true radius vector, and true longitude, of the 'centre of gravity of Earth and Moon,' measured from the Sun; a,′ r', v', those of the Moon as measured from the Earth; $v_{,}$ the tropical longitude of the Moon; l′, the latitude of the Moon. The perturbing forces are P, measured from the Earth in the projection of the Moon's radius vector on the fixed plane; T, at right angles to P in that plane, accelerating the tangential motion; and Z, measured from that plane and at right angles to it.

V. The three gravitational equations of motion are the following: in which the left-hand-side contains nothing but ordinates and forces produced by the Earth and Moon considered as points, and the right-hand-side contains nothing but forces produced by the Earth's oblateness and the Sun's disturbance (applicable in the same shape to a planet's disturbance):—

$$\frac{a'^3}{\epsilon'+\mu'}\cdot\left(\frac{a}{a'}\right)^2\cdot\frac{d}{dt}\left\{\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos\mathrm{l}'\right)^2\cdot\frac{dv'}{dt}\right\}=\frac{a'^2}{\epsilon'+\mu'}\left(\frac{r'}{a'}\cos\mathrm{l}\right)\cdot\mathrm{T}.$$

$$\frac{a'^3}{\epsilon'+\mu'}\cdot\frac{d}{dt}\left\{\left(\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos\mathrm{l}'\right)\right)^2+\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos\mathrm{l}'\cdot\frac{dv'}{dt}\right)^2\right\}$$

$$+2\,\frac{a'}{a}\cdot\left(\frac{a'}{r'}\right)^3\cos\mathrm{l}'\cdot\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos\mathrm{l}'\right)$$

$$=2\,\frac{a'^3}{\epsilon'+\mu'}\,\frac{\mathrm{P}}{a}\cdot\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos\mathrm{l}'\right)+2\,\frac{a'^3}{\epsilon'+\mu'}\cdot\frac{\mathrm{T}}{a}\cdot\frac{a'}{a}\cdot\frac{r'}{a'}\cos\mathrm{l}'\frac{dv'}{dt}\cdot$$

$$\frac{a'^3}{\epsilon'+\mu'}\cdot\frac{d^2}{dt^2}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cdot\sin\mathrm{l}\right)+\frac{a'}{a}\left(\frac{a'}{r'}\right)^2\sin\mathrm{l}=\frac{a'^3}{\epsilon'+\mu'}\cdot\frac{\mathrm{Z}}{a}\cdot$$

VI. For developing the left-hand-sides by substitution of Delaunay's values (subsequently to be furnished with additional symbols of contingent corrections), Delaunay's coefficients are to be converted into simple numbers: those for the powers of $\frac{r}{a}$ or $\frac{a}{r}$ into fractions of unity in which the last retained unit is the ten-millionth part of unity or 10^{-7}; and those for the terms of longitude and latitude into fractions of radius in which the last-retained unit is the ten-millionth part of radius (corresponding very nearly to 0″·02). The decimal cyphers preceding the efficient figures are to be omitted.

VII. For developing the right-hand-sides, it will be necessary to adopt for the last-retained unit the thousand-millionth part of unity or radius, in order to secure the terms which rise greatly by integration. But this amounts practically to the same extent of development as that already mentioned, because two decimal places are supplied by the smallness of the external factor representing the Sun's disturbing force (the value of its principal term being about $\frac{1}{180}$).

VIII. The following appears to be the expression for the right-hand-side of the first equation; where e is the ellipticity of the Earth's surface; $\frac{eq.\ c.\ f.}{grav.}$ is the proportion of centrifugal force at the equator to gravity; c is the Earth's polar semi-axis, and ω is the inclination of the equator to the ecliptic:—

$$+\left[\left(e-\frac{eq.\ c.\ f.}{2\ grav.}\right)\cdot\left(\frac{c}{a'}\right)^2\right]\left(\frac{a'}{r'}\right)^3\cdot\Big\{-2\sin^2\omega\cdot\cos^2 l\cdot\sin v_{,}\cdot\cos v_{,}$$
$$-2\cos\omega\cdot\sin\omega\ \cos l\cdot\sin l\cdot\cos v_{,}\Big\}$$

$$+\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\left(\frac{A'}{R'}\right)^3\cdot\left(\frac{r'}{a'}\right)^2\cdot\Big\{-3\cos^2 l\cdot\cos\overline{|v'-V'|}\cdot\sin\overline{|v'-V'|}\Big\}$$

$$+\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\cdot\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^3\cdot\Big\{+\frac{15}{2}\cos^3 l\cdot\cos^2\overline{|v'-V'|}\cdot\sin\overline{|v'-V'|}$$
$$-\frac{3}{2}\cos l\cdot\sin\overline{|v'-V'|}\Big\}$$

$$+\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^3+\mu^3}{(\epsilon+\mu)^3}\cdot\left(\frac{a'}{A'}\right)^2\right]\left(\frac{A'}{R'}\right)^5\cdot\left(\frac{r'}{a'}\right)^4\Big\{-\frac{35}{2}\cos^4 l\cdot\cos^3\overline{|v'-V'|}\cdot\sin\overline{|v'-V'|}$$
$$+\frac{15}{2}\cdot\cos^2 l\cdot\cos\overline{|v'-V'|}\cdot\sin\overline{|v'-V'|}\Big\}$$

$$+\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\left(\frac{A'}{R'}\right)^3\cdot\left(\frac{r'}{a'}\right)^2\cdot bt\Big\{-3\cos l\cdot\sin l\cdot\sin\overline{|v'-V'|}\cdot\sin\overline{|V'-K|}\Big\}$$

$$+\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu^2}\cdot\frac{a'}{A'}\right]\cdot\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^3\cdot bt$$
$$\times\Big\{+15\cos^2 l\cdot\sin l\cdot\cos\overline{|v'-V'|}\cdot\sin\overline{|v'-V'|}\cdot\sin\overline{|V'-K|}\Big\}$$

The first line contains the effect of the Earth's oblateness; the second line contains the solar perturbations which first present themselves; the third line contains the principal parallactic terms; the fourth line, succeeding terms of smaller magnitude; the fifth and sixth lines, terms depending on the secular change in the position of the ecliptic, whose effect on the heliocentric latitude of the 'centre of gravity of Earth and Moon' is supposed to be represented by the term $bt\sin\overline{|V'-K|}$

A general periodical term may be added, to represent perturbation produced by any unrecognised cause.

IX. The right-hand-side of the second equation appears to consist of the following terms:—

$$+2\left[\left(e-\frac{eq.\,c.\,f.}{2\,grav.}\right)\left(\frac{c}{a}\right)^2\right]\left(\frac{a'}{r'}\right)^4\cdot\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos l'\right)$$

$$\times\left\{\begin{matrix}5\,(\sin\omega\cdot\cos l\cdot\sin v_{,}+\cos\omega\cdot\sin l)^2\cdot\cos l\\-2\,(\sin\omega\cdot\cos l\cdot\sin v_{,}+\cos\omega\cdot\sin l)\cdot\sin\omega\cdot\sin v_{,}\\-\cos l\end{matrix}\right\}$$

$$+2\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')\cdot A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\cdot\left(\frac{A'}{R'}\right)^3\frac{r'}{a'}\cdot\frac{d}{dt}\left(\frac{a'}{a}\,\frac{r'}{a'}\cos l'\right)$$

$$\times\left\{+3\cos l'\cdot\cos^2\overline{|v'-V'|}\;-\cos l'\right\}$$

$$+2\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')\cdot A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^2\cdot\frac{d}{dt}\left(\frac{a'}{a}\,\frac{r'}{a'}\cos l'\right)$$

$$\times\left\{-\frac{15}{2}\cos^2 l'\cdot\cos^3\overline{|v'-V'|}+\frac{3}{2}\cos\overline{|v'-V'|}+3\cos^2 l'\cdot\cos\overline{|v'-V'|}\right\}$$

$$+2\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')\cdot A'^3}\cdot\frac{\epsilon^3+\mu^3}{(\epsilon+\mu)^3}\cdot\left(\frac{a'}{A'}\right)^2\right]\cdot\left(\frac{A'}{R'}\right)^5\cdot\left(\frac{r'}{a'}\right)^3\cdot\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos l'\right)$$

$$\times\left\{+\frac{35}{2}\cos^3 l\cdot\cos^4\overline{|v'-V'|}-\frac{15}{2}\cos l\cdot\cos^2\overline{|v'-V'|}\right.$$

$$\left.-\frac{15}{2}\cos^3 l\cdot\cos^2\overline{|v'-V'|}+\frac{3}{2}\cos l\right\}$$

$$+2\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\cdot\left(\frac{A'}{R'}\right)^3\cdot\frac{r'}{a'}\cdot\frac{d}{dt}\left(\frac{a'}{a}\,\frac{r'}{a'}\cos l'\right)$$

$$\times b\,t\cdot\left\{+3\sin l'\cdot\cos\overline{|V'-K|}\;\sin\overline{|V'-K|}\right\}$$

$$+2\left[\frac{\sigma'\cdot a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\cdot\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^2\cdot\frac{d}{dt}\left(\frac{a'}{a}\,\frac{r'}{a'}\cos l'\right)$$

$$\times b\,t\left\{-15\cdot\cos l'\cdot\sin l'\cdot\cos^2\overline{|v'-V'|}\cdot\sin\overline{|V'-K|}+3\cos l'\cdot\sin l'\cdot\sin\overline{|V'-K|}\right\}$$

$$+2\frac{dv'}{dt}\times\left\{\text{all the terms of Article VIII.}\right\}$$

X. The right-hand-side of the third equation appears to consist of the following terms:—

$$+\left[\left(e-\frac{eq.\,c.\,f.}{2\,grav.}\right)\left(\frac{c}{a}\right)^2\right]\left(\frac{a'}{r'}\right)^4$$

$$\times\left\{\begin{matrix}5\,(\sin\omega\cdot\cos l\cdot\sin v_{,}+\cos\omega\cdot\sin l)^2\times\sin l-2\cdot\sin\omega\cdot\cos\omega\cdot\cos l\cdot\sin v_{,}\\-\sin^2\omega\cdot\sin l-3\cos^2\omega\cdot\sin l\end{matrix}\right\}$$

$$+\left[\frac{\sigma' a'^3}{(\epsilon'+\mu')\cdot A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\left(\frac{A'}{R'}\right)^3\cdot\frac{r'}{a'}\cdot\left\{-\sin 1\right\}$$

$$+\left[\frac{\sigma' a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\cdot\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^2\left\{+3\cdot\cos 1\cdot\sin 1\cdot\cos\overline{|v'-V'|}\right\}$$

$$+\left[\frac{\sigma' a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^3+\mu^3}{(\epsilon+\mu)^3}\cdot\left(\frac{a'}{A'}\right)^2\right]\cdot\left(\frac{A'}{R'}\right)^5\cdot\left(\frac{r'}{a'}\right)^3$$

$$\left\{-\frac{15}{2}\cos^2 1\cdot\sin 1\cdot\cos^2\overline{|v'-V'|}+\frac{3}{2}\sin 1\right\}$$

$$+\left[\frac{\sigma' a'^2}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\left(\frac{A'}{R'}\right)^3\cdot\frac{r'}{a'}\cdot bt\left\{+3\cos 1\cdot\cos\overline{|v'-V'|}\cdot\sin\overline{|V'-K|}\right\}$$

$$+\left[\frac{\sigma' a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\cdot\left(\frac{A'}{R'}\right)^4\cdot\frac{r'^2}{a'}\cdot bt$$

$$\times\left\{\begin{array}{l}-\frac{15}{2}\cos^2\lambda\cdot\cos^2\overline{|v'-V'|}\cdot\sin\overline{|V'-K|}\\ +\frac{3}{2}\sin\overline{|f-V'|}+3\sin^2 1\cdot\sin\overline{|V'-K|}\end{array}\right\}$$

All these algebraical expressions require revision, and may perhaps admit of some reduction.

XI. The nature of the considerations which will introduce some of the contingent variations may be judged partly from discussions similar to the following:—

Assuming that the length of the mean sidereal year is invariable, and is accurately known, $\frac{\sigma'}{A'^3}$ is invariable, and is a known quantity. But A′ is not certainly known; and, therefore, if A be its assumed value, we must write its real value in some such shape as this: $A' = A\left(1 + (1)\right)$, the quantity (1) being a contingent correction. Then, as $\frac{\sigma'}{A'^3}$ is invariable, or $= \frac{\sigma}{A^3}$, $\sigma' = \sigma\frac{A'^3}{A^3} = \sigma\left(1 + 3\,(1)\right)$.

Now we cannot assume that the length of the mean sidereal month is invariable; moreover, our estimate of the masses $\epsilon + \mu$ may be liable to error. Therefore, a' ought to be expressed by $a\left(1 + (2)\right)$, where a is the assumed value for 1900, and where (2) contains a constant value, and also a value depending on time. And $\epsilon' + \mu'$ must be represented by $\left(\epsilon+\mu\right)$

$\left(1+(3)\right)$, where (3) is constant. Therefore, $\frac{\sigma' a'^3}{(\varepsilon'+\mu')A'^3}$ will become $\frac{\sigma a^3}{(\varepsilon+\mu)A^3} \times \left(1+3(1)\right) \cdot \left(1+3(2)\right) \cdot \left(1-(3)\right) \cdot \left(1-3(1)\right) = \frac{\sigma a^3}{(\varepsilon+\mu)A^3} \times \left(1+3(2)-(3)\right)$. And to form $\frac{\sigma' a'^3}{(\varepsilon'+\mu')A'^3} \cdot \frac{a'}{A'}$, we must multiply the last expression by $\frac{a}{A}\left(1+(2)-(1)\right)$, and it becomes $\frac{\sigma a^3}{(\varepsilon+\mu)A^3} \cdot \frac{a}{A} \times \left(1-(1)+4(2)-(3)\right)$.

No general rule can be given for the appropriation of contingent symbols to terms. But every principal coefficient (as the coefficients of elliptic equation, evection, variation, annual equation, inclination, evection in polar distance, and many much smaller coefficients,) must have its independent symbol of contingent correction.

The first part of the process of applying these principles to the Lunar Theory will consist in developing the expressions of Articles VIII., IX., X., by substitution in them of Delaunay's numerical values of $\frac{a}{r'}$, v, l, and in ascertaining how nearly these values will satisfy the three equations of Article V.

I have developed, numerically, the expressions for all the requisite powers of $\frac{r}{a}$ and $\frac{a}{r}$, and I am proceeding with the development of sin l and various powers of cos l.

1874, *January* 9.

Additional Notes concerning Sir William Herschel's Double Stars.

By S. W. Burnham, Esq.

(*Communicated by Mr. Dunkin.*)

I have given some attention, more particularly within the last few months, to the telescopic examination of Double Stars with reference to which there exists some uncertainty as to place, magnitude, position-angle, and distance, identity with other double stars, &c. There is a large number of such objects, several hundred perhaps, principally from the Catalogues of the two Herschels, which require or may require correction in some of these respects. Some have been found to be out of their

assigned places, and very many ascertained to be identical with stars heretofore regarded as distinct objects.

At the time of preparing the paper on "Errors and Omissions in the Catalogue of Sir William Herschel's Double Stars" (*Monthly Notices*, xxxiii. No. 9, Supplementary Number), I was not aware of a communication on the same subject by Sir John Herschel (*Monthly Notices*, xxviii. p. 151), so that a few of my corrections had already been noted.

The following, some of the results of an examination or search for the stars referred to, relate for the most part to their identity. In a few instances, as will be seen, a considerable error in place has been detected. All the places given from Herschel's Catalogue (*Memoirs of the R. A. S.*, xxxv.), or otherwise, are for 1880.—

H. I. 47.

No measures of distance: P=336°·8. My estimates are: P=330°: D=$2\frac{1}{2}''$: Mags. 8, $8\frac{1}{4}$. The place of this star from the *Washington Transit Zones* (No. 9, Z. 205) makes Herschel's declination 7′ too large. After these observations, I found that this pair was identical with H. 5252 (*Cape Observations*), where P=328°·0: D=2″·1: Mags. 8, 8. The place from *Cape Observations* agrees exactly with that from the *Washington Zones*.

H. I. 60.

P=286°·8. No measures of distance. Herschel II. says, "It is barely possible that this may be Σ 2441 with a change of 5° in position, but the place disagrees by 18′ in P. D." I have not been able to find any pair nearer than Σ 2441, which answers the description. H. 1366 has the same declination as H. I. 60, but is about 3^m *p*. This is a very faint pair, and not likely to be Herschel's, even if the position-angle corresponded. (P=57°·8: D=10″±: Mags. 9·10, 12). About $2\frac{1}{2}^m$ *p*, the last is an 8 mag. star with a distant, very minute double companion. There is very little, if any, doubt of the identity of H. I. 60 with Σ 2441.

H. I. 77.

No measures of distance are given by Herschel. He makes P=7°·6. I have looked up this pair, a very pretty double, and estimate D=2″: P=10°.

H. I. 90.

I have carefully examined the vicinity, and cannot find any double of Class I. in this place, and no pair nearer than S. 788 and Σ 2781.

H. III. 20.

P=329°·5: D=7″·6. "Identification very precarious from vagueness of description." Although the place given is 2^m *p*, and 24′ *n* of Σ 1838, they are undoubtedly the same. Struve gives: P=333°·8: D=9″·04.

H. III. 42.

No measures in Herschel. Σ 246 is $3^m f$ and $5' n$, and the only pair to be found in the immediate vicinity.

H. III. 52.

P=52°·9 : D=13″·67. "Identification precarious; by bare possibility S. 465." This is certainly Σ 630 (=S. 465). Struve gives: P=49°·2 : D=14″·0. Herschel's place should be diminished 25^s in R.A., and 28′ in Declination.

H. III. 103.

P=148°·4 : D=12″·97. This is Σ 2446 (=P. xviii. 302) where P=154°·5 : D=10″·13. There is a second minute companion, not mentioned by either, in the opposite direction, at a distance of 30″ or 40″. Herschel's place agrees substantially with Struve's.

H. IV. 34.

D=30″±. "Near 64 *Aquilæ*. No other indication of place." The only pair found is one in the field with *n p* 64 *Aquilæ*. P=190° : D=30″ or more : Mags. 9, 10.

H. IV. 127.

P=339°·9 : D=16″·55. In the Catalogue this is called Piazzi xviii. 274 (that star being Σ 2434), and in his list of corrections Sir John Herschel states explicitly that H. IV. 127 is Σ 2434 (=Sh. 285), the place of which had been given in the first instance. Struve's measures of the wide pair of that star are: P=147°·0 : D=25″·56. Suspecting from the discordance between the measures of Herschel and Struve, that these two stars were not the same, I carefully examined all the stars in the vicinity, and found beyond all question that Herschel's pair was identical with Σ 2447. Struve gives: P=334°·9 : D=13″·82, differing but slightly from the measures of Sir W. Herschel quoted above. The place of H. IV. 127 should therefore be increased 3^m 48^s in R.A. and 39′ in Dec.

H. V. 31.

D=30″. "A choice between two stars, but the preceding the most probable." There are several small stars near this place answering the description in respect to distance, but the absence of the position-angle renders any absolute identification impossible.

H. V. 103.

"The place that of a star 8 m. in Argelander, very unequivocally indicated." This star is L. 35845, the place of which differs but slightly from Herschel's. It is also correctly given in *Positiones Mediæ*.

H. VI. 47.

R.A $=19^h\ 19^m{\cdot}9$; Dec.$=+1°35'$. No measures. "Identification very precarious." The only pair found is L. 36616, of which P$=105°$: D$=60''$: Mags. $7\frac{1}{2}$, $8\frac{1}{2}$. Reduced from Lalande, R.A.$=19^h\ 18^m\ 49^s$: Dec. $+1°36'$.

H. VI. 48.

R.A.$=19^h\ 20^m$: Dec.$=+1°20'$. No measures. "Identification very precarious." The only double found in the vicinity besides the last is L. 36659. P$=340°$: D$=25''$: Mags. 9, 11. The place from Lalande is: R.A.$=19^h\ 20^m\ 17^s$: Dec.$=+1°33'$.

H. VI. 49 and 50.

No measures given of these stars, which have the same approximate R.A. with a difference of but 5′ in Dec. One of them must be P. xviii. 197, of which Smyth's measures are: P$=168°{\cdot}9$: D$=99''{\cdot}0$: Mags. 7, 9.

H. N. 10.

This star is entered, without measures, as Class III. with the position given as "preceding." Sir John Herschel has given no synonyms, and yet this pair is found in no less than four other double star Catalogues, two of which are by Sir John Herschel himself. The corresponding numbers are: H. N. 10$=\Sigma$ C. P. 693 $=$S.763$=$H. 2996$=$H. 3002. South gave: P$=295°{\cdot}1$: D$=16''{\cdot}75$, and Herschel (2996) P$=291°{\cdot}6$: D$=15''\pm$. The places agree, with the exception of H. 3002, which is about 4^m preceding in R.A. There Herschel gives P$=120°\pm$: D$=25''\pm$, which he states were "estimated from a diagram." This will probably explain the reversing of the angle, and the increased distance. He calls it "a very fine double star," but gives no magnitudes. I have very carefully examined on several occasions all the stars within a considerable radius, and I am absolutely certain that these various Catalogues all refer to the same pair. There is something strange about the great difference in the estimated magnitudes by different observers. In the B.A.C. (7202) it is 6 m.; South, $7\frac{1}{2}$, 8; Herschel (2996) 9, 9+; in the *Washington Transit Zones*, where it is noted as "a beautiful double star," it is given 9 m. When examined recently the magnitudes appeared to be about as set down by South. In a low-power field 8′ directly south I noticed a pretty 2″ pair of very minute stars.

H. N. 18.

No measures in Herschel, Class II. A rather faint pair, estimated: P$=250°$: D$=5''$: Mags. 9, 9+.

H. N. 35.

No measures; angle "following." Herschel's place is:

R.A.=23^h $31^m{\cdot}3$: Dec.=$-13°$ 45′. This is *Aquarii* 355, and its place from Lalande (46271): R.A.=23^h 31^m 27^s: Dec. =$-13°$ 43′ 34″. Estimated, P=90°: D=40″: Mags. 7, 10.

H. N. 39.

R.A.=21^h $14^m{\cdot}2$; Dec.=$+39°$ 13′: P=120°·5: D=18″± "Place that of the most probable star in Argelander." This is identical with No. 648 of Sir John Herschel's 7-ft. equatoreal measures (*Memoirs of the R. A. S.* xiii.), where it is marked "nova." There P=114°: D=26″·45: Mags. 7, 11. I find that this is Weisse (2) xxi. 312, from which R.A.=21^h 14^m 17^s: Dec. =$+39°$ 15′. It is about 1^h 20^s following Σ 2785.

H. N. 63.

R.A.=14^h $55^m{\cdot}6$: D=$+54°$ 15′. This is Σ C. P. 470; and the place, from *Positiones Mediæ*, R.A. = 14^h 56^m 0^s: Dec.=+ 54° 20′.

H. N. 101.

Class III. No measures. Found and estimated, P=330°: D=10″: mags. $8\frac{1}{2}$, 9.

H. N. 102.

P=300; Class III. In or very near this place is a pair of 10 m. stars. P=300°: D=5″. I failed to find any other double at all corresponding.

H. N. 110.

Class V. No measures. R.A. = 19^h 41^m 48^s: Dec. = + 32° 47′. Herschel's declination is too large, as this is S. 726, the place of which, from *Positiones Mediæ*, is:—R.A.= 19^h 41^m 59^s: Dec.=$+32°$ 36′.

H. N. 112.

No measures. The polar distance in the Catalogue is 19° 4′, which, as corrected by Herschel, should be 99° 4′. This makes it identical with Σ 3008 (= S. 829), the places agreeing very nearly. This pair is now a well-recognised binary. It is B.A.C. 8154.

H. N. 127.

The place given is the same as that of H. VI. 4 (a^2 *Capricorni*), while H. VI. 92, which is described as in or near the place of H. N. 127, is 30′ north and 5^m preceding. Referring to Sir William Herschel's original Catalogue, I find this designated as "the middle one of 3, *n f a Capricorni*."

H. N. 129.

R.A.=18^h $57^m{\cdot}5$: Dec.=$-22°$ 54′·9. No measures; Class I. This is L. 35530, from which its place is:—R.A. = 18^h 56^m 0^s: Dec.=$-23°$ 4′ 31″. My estimates of angle and distance are as follow:—P=300°: D=8″: Mags. $7\frac{1}{2}$, 10.

H. N. 130.

Class I. I have not been able to find any double star of the first class in this place.

H. N. 131.

"Unidentifiable; Class III." R.A. = $21^h\ 49^m \pm$: Dec. = $-15°\ 6' \pm$. Although the difference in declination is considerable, I have no doubt, after a careful search, that this pair is the same as H. 3071, the place of which is:—R.A. = $21^h\ 51^m\ 14^s$: Dec. = $-15°\ 42'$. Herschel gives of this, P = 318°·5 : D = 18″ : Mags. 8, 11. "Fine." There is a 20″ pair of 12 m. stars (H. 5522, *Cape Observations*) very near the place of H. N. 131; but from the faintness of both stars it is too insignificant an object to make it at all probable that it is the pair discovered by Herschel I.

H. N. 139.

R.A. = $21^h\ 12^m{\cdot}1$: Dec. = $-15°\ 48'$. No measures; Class I. This place is also largely in error if the double found is the one in question. The only double of Class I. found after a careful search and examination of all the stars near, is L. 41483. It is a beautiful and interesting pair. My estimates of position and distance are:—P = 100°: D = 1″·5: Mags. 8, 10. Its place from Lalande is:—R.A = $21^h\ 15^m\ 39^s$: Dec. = $-15°\ 26'$.

H. N. 140.

R.A. = $22^h\ 37^m{\cdot}1$: Dec. = $-5°\ 30'$. Class II. I have not examined this star. Struve, in *Positiones Mediæ*, gives its place:—R.A. = $22^h\ 35^m\ 51^s$: Dec. = $-5°44'$, which is probably correct.

Chicago, U.S., 1873, *December* 8.

On Red Stars in Cygnus. By G. F. Chambers, Esq.

In Vol. xxxiv. of the *Monthly Notices*, p. 54, I observed some remarks by Mr. J. Birmingham concerning Red Stars catalogued by Schjellerup, on which I should like to say a few words, for Red Stars have long interested me, and I have paid much attention to them. It was on this account that I incorporated Schjellerup's list into my *Descriptive Astronomy*, making however some additions to it. After the publication of the Catalogue in that work, I resolved upon undertaking an examination on my own account of every star contained therein which was visible in England, with

the ultimate object of eliminating all stars which were beyond the reach of a 4-inch refractor, or otherwise were devoid of interest.

On reading Mr. Birmingham's paper, I turned to my annotated Catalogue of Red Stars, and there found two stars apparently identical with those alluded to by Mr. Birmingham, and by him designated, from Schjellerup, Nos. 251 and 252.

In the Catalogue in my book they are respectively numbered 264 and 265, and their places for 1870, and other particulars, are set down as follow :—

	R.A. h m s	Decl. ° ′	Mag.	Remarks.
264. Anon. Cygni	21 37 54	+37 25·3	8	Red.
265. Anon. Cygni	21 38 59	+37 16·1	8·5	Extremely intense ruby.

It was in 1869 that I began to examine all the Red Stars in the aforesaid Catalogue, and I ceased work in the summer of 1871. All the stars which I viewed were one by one marked in classes as follow :—

1. Conspicuously red, and altogether very interesting.
2. Tolerably red : worth looking at.
3. Reddish : not worth much.
4. More or less devoid of colour, and to be removed from the Catalogue in consequence.

In addition to the above classification, the remarks attached to each star were considered, and if thought desirable, revised.

From my MS. notes I find that at some period between the summer of 1869 and that of 1871, I looked for and found *both* the stars of which one is now understood to be missing. The column of remarks I find I did not alter, but I put my 264th star into the 2nd class, as above, and my 265th star (the missing one) into the 1st class. Both stars I have no doubt were viewed on the same evening, and probably in the order of R. A., that being my invariable custom in such matters.

What the value of this testimony may be I will not presume to assert, but having regard to the circumstances under which I worked, I do not think I could have fallen into a mistake.

The telescope employed was a 4-inch Cooke refractor, equatoreally mounted in a fixed observatory in Kent, where I was residing during the years in question.

Eastbourne, Sussex,
1874, *January* 8.

On a reported Occultation of Regulus by the planet Venus, A.D. 885, September 9. By J. R. Hind, Esq.

In the manuscript of the work of the Arabian astronomer Ibn-Jounis, described by Delambre, *Astronomie du moyen Age*, p. 76, *Regulus* is said to have been occulted by *Venus*, on a date which corresponds to A.D. 885, September 9, in the Julian Calendar. The observation is thus given at p. 87: "Occultation de *Régulus* par *Vénus*, le 9 Septembre 885. Une heure avant le lever du Soleil, chacun des deux jours précédens, le mouvement de *Vénus* avait été de plus de 1°."

I have examined this observation by means of M. Le Verrier's Tables of the Sun and Planets, and with the following results. I give my numbers in sufficient detail to admit of verification by any one curious enough to repeat the work:—

A. D. 885.	Paris M. T. d h	♀ Helioc. long. ° ′ ″	♀ Helioc. lat. ° ′ ″	♀ Rad. Vect.	☉ True long. ° ′ ″	☉ Log. Rad. Vect.
Sept.	9 12	77 12 7	+0 37 32	0·7191606	351 23 14	0·0002061
,,	10 0	78 0 40	0 40 21	0·7191138	351 52 47	0·0001429
,,	10 12	78 49 12	+0 43 8	0·7190670	352 22 20	0·0000795

Hence the geocentric places of *Venus* are:

	d h	True R. A. ° ′ ″	True Decl. ° ′ ″	Log. Distance.
Sept.	9 12	136 53 30	+17 0 35	0·07509
,,	10 0	137 29 39	16 51 41	0·07626
,,	10 12	138 5 45	+16 42 38	0·07741

Aberration in R.A. . . .	−29″·7	Diameter of *Venus*	14″·0
,, in Decl. . . .	+ 7·4	Horizontal Parallax of *Venus*	7·5

I take the position of *Regulus* from the last Greenwich Catalogue (1864), and its proper motions from M. Le Verrier's *Annales*, II. p. 198, and by the rigorous trigonometrical formulæ thus find for the beginning of A.D. 885:

Regulus	Mean R. A. .	136 59 3·7	Mean δ +17 0 58·0

The reduction constants after Bessel for the date are:

log. A +1·2672 log. B −0·4860 log. C +9·5634 log. D −0·4312

and, therefore, the corrections for apparent place of star:

R. A. +2″·6 Decl. +1″·2

I assume the observation to have been made at Baghdad; and from the above numbers find the conjunction in Right Ascension would take place on September 9, at $16^h\ 50^m$ mean time, the planet being then $1'\ 47''$ south of the star—both geocentric. The nearest approach would occur about $16^h\ 43^m$, at which time the distance may be taken $1''.7$. The Sun rose at Baghdad at $17^h\ 42^m$; and the hour before sunrise, when it would appear the observation was made, therefore corresponds to the time found for nearest approach.

At a distance of $1''.7$ it may perhaps be granted that the star would be lost to the naked eye in the comparative blaze of the planet, and thus M. Le Verrier's Tables sufficiently represent the place of *Venus* one thousand years ago. It will be remarked, that this near approach took place when the planet was in nearly the same part of its orbit that it will occupy during the approaching transit.

As an addendum to the above, I give the results of my examination of another interesting observation recorded by the same Arabian astronomer (Delambre, *Astr. du moyen Age*, p. 80): "Conjonction de ♂ et ♀, 13 Février 864. ♀ et ♂ paraissaient à la vue se toucher au commencement de la nuit." I again make use of M. Le Verrier's Tables of the Sun and Planets, and find:

A. D. 864.	Paris M. T. d	h	☉ True long. °	′	″	☉ Log. Rad. Vect.
Feb.	13	0	148	43	32·9	9·9967588
„	13	6	148	58	33·2	9·9967883

	d	h	♀ Helioc. long. °	′	″	♀ Helioc. lat. °	′	″	♀ Log. Rad. Vect.
Feb.	13	0	52	44	21·2	−0	47	49·9	9·8578530
„	13	6	53	8	30·8	−0	46	26·6	9·8578329

	d	h	♂ Helioc. long. °	′	″	♂ Helioc. lat. °	′	″	♂ Log. Rad. Vect.
Feb.	13	0	24	50	5·3	−0	30	25·3	0·1654763
„	13	6	24	58	35·3	−0	30	9·0	0·1655668

And hence the geocentric places:

	d	h	♀ Long. °	′	″	♀ Lat. °	′	″	♂ Long. °	′	″	♂ Lat. °	′	″
Feb.	13	0	2	36	9	−0	26	49	2	37	1	−0	20	26
„	13	6	2	54	17	−0	26	4	2	48	6	−0	20	15

The conjunction in Longitude would therefore occur February $13^d\ 3^h\ 32^m$ mean time at Baghdad, when *Mars* would be north of *Venus* $6''.3$, which is a degree of approximation not inaptly represented by the words of the Arabian astronomer, as regards naked-eye observation.

On the present Dimensions of the White Spot Linné. By C. E. Burton, Esq.

On the 4th of June 1873, the length of the major axis of the white cloud was measured by my friend Mr. Erck, and the mean of the measures was found to be 4″. Instrument Alvan Clark's $7\frac{1}{2}$-inch. Power 200. The terminator passed at the time close to the eastern walls of *Copernicus* and *Clavius*. On the same night I made a sketch of *Linné* with my own 12-inch reflector, using powers of 238 and 408.*

During the next lunation another sketch was obtained with the same instrument on July 15 at 15^h, approximate G. M. T., at which time the terminator had just passed *Bessel*. On this occasion no trace of the white spot was seen, though the definition was very fine, and it was carefully looked for. The small crater of *Linné* was beautifully shown. Instrument used, the 12-inch equatoreal reflector; powers 238, 514, and 940; all measured, and all single lenses.

1873, December 27 at $6^h\ 15^m \pm$ G. M. T. Measured the major diameter of the white spot. Mean of six measures 5″·5. Extreme difference of measures 0″·98. Terminator touching east wall of *Eratosthenes* and west wall of *Clavius*. The shadow thrown by the west wall of the small crater internally was well seen at times with a positive power of about 240. No visible shadow from east wall of craterlet.

1873, December 31. Time not recorded, but about 9^h G. M. T. The major axis of the white spot was measured with the position-wire-micrometer; power 240. 10 measures. Mean 5″·45. Extreme difference 0″·82. Three measures of minor axis taken G. D. 0″·48. Mean 3″·4.

1874, January 2 at $11^h \pm$ G. M. T. Four measures. Mean 5″·3. G. D. 1″. Power 240. One measure with 120 gave 5″·4 for the same dimension.

1874, January 6 at $12^h\ 30^m \pm$ G. M. T. Definition indifferent, and wind troublesome. Mean of five measures 4″·8. Power 240. Terminator close to west boundary of the *Mare Crisium*.

On December 31, six additional measures of the major axis of the white spot were taken with power 400, with which the edges of the spot appeared extremely nebulous.

Similarly, on January 2, 400 gave the spot a diameter of 4″, the measures being accompanied by the note, that the whiteness was very ill-terminated.

High Illumination.—*Linné* appears under good definition as a white spot, the boundary of which is tolerably distinct, surrounded by a nebulous chevelure, of varying width, which fades rapidly into the surrounding tint. When definition is not at the best, and the power used is high, the inner white spot will be measured,

* The two sketches made by Mr. Burton were exhibited at the meeting.

the terminal nebulosity not attracting attention. This probably is the explanation of the comparatively small diameters given by power 400.

On tabulating the observations of the white spot made by Dr. Huggins, Professor Tacchini, Mr. Erck, and myself, we have:

Date.	Diameter.	Observer.	
1867, July 9	7″·85	Dr. Huggins.	Moon 8 days old.
1867,	7 ·94	Prof. Tacchini	
1873, June 4	4 ·0	Mr. Erck.	
1873, Dec. & Jan.	5 ·4	B.	Mean of the three best results with the 12-in. reflector.

Is not the white spot in a state of change?

Loughlinstown, Co. Dublin,
1874, *January* 7.

Note on the Lunar Crater Linné. By William Huggins, Esq., D.C.L., LL.D., F.R.S.

The different aspects which this crater assumes under different conditions of illumination have raised the question whether any permanent change can be ascertained to have taken place in it.

In 1867, I presented to the Society a Note on the appearance of this object, and some micrometrical measures of the Bright Spot and of the small Crater. (*Monthly Notices*, vol. xxvii. p. 296.) In that Note I referred to some early observations of the object which appeared to me to be in favour of the unaltered state of the crater from the time of Schroter (1788) to that of the observations of 1867.

I venture now to present some recent observations and measures of *Linné*, as possibly of some value in reference to the question, whether during the last eight years we have any indication of permanent change.

The White Spot.—Under some conditions of illumination, the shallow saucer-like form of the large crater is seen distinctly; at other times the surface of the White Spot is similar in appearence to a cloud, presenting no distinct details and remaining undefined when the small neighbouring craters are seen with distinctness.

This cloud-like appearance is strong on the south-east and north-west sides of the shallow crater; and when the form of this

crater is not seen, this cloud-like appearance is so marked on these sides as to change the apparent direction of the oval form of the spot. At these times, the long axis of the oval appears to be in the direction from E.S.E. to W.N.W., but the true direction of the longer axis of the shallow crater is from S.E. to N.W.

November 5, 1867, I found this long axis to be inclined to a parallel of declination at an angle of 55°.

Two measures, on December 27, 1873, gave an angle of 48°. This difference is not greater than may be due to the circumstance that, in trying to place the wire of the micrometer in the direction of the longer axis, the judgment is probably influenced by the side of the crater which is seen with greater distinctness, and as the object is not a regularly formed oval, it seems not improbable that some discrepancy may present itself from this cause between measures taken at different times.

In my former Note on *Linné* I gave the following measures of the Bright Spot:—

July 9, 1867	Length of Spot	7″·85
	Breadth „	6·14

The crater was measured again on

November 5, 1867	Length	7″·58
	Breadth	5·95
December 27, 1873	Length	7·16
	Breadth	4·3

I would remark, that in the breadth of crater, that is the direction from S.E. to N.W., the measures are more likely to vary, because, as I have already remarked, it is on these sides that the outline of the crater is more interrupted by the cloud-like appearance of the surface, and changes more quickly by differences of illumination. At the time of making these measures, and almost before completing them, I lost the distinct vision of the outline of the shallow crater in the white cloud-like appearance which came over the spot.

Small Crater.—The words I used in my Note of 1867 describe exactly the appearance of this object on December 27, 1873: "In the centre of *Linné*, but rather nearer to the western margin, was seen the small crater. This object was well defined in the telescope. The interior of the small crater was in shadow, with the exception of a small part of it towards the east." "The small crater, which appears to be deep, has a narrow margin brighter than the White Spot on which it falls. The measures of the crater include the narrow bright margin."

In 1867, I measured the small crater across in several directions, and then took the mean of the measures on the sup-

position that the small crater was sensibly round. The result was: Diameter of small crater, 1″·71.

In my recent measures, I took one set in the direction of the longer axis of the large crater, and a second set at right angles to that direction. These measures seem to show that the small crater did not appear quite round at this time, but was sensibly oval in a direction at right angles to the larger crater. I give all the readings of the micrometer, that a better estimate may be formed of the value to be given to the measures.

First set, in the direction of the longer axis of the shallow crater :

21·5
21·
21·
—— mean 21· = 1″·73

Second set, at right angles to the former direction:

22·5
22·7
23·
—— mean 22·7 = 1″·98

The mean of these, 1″·8, agrees almost exactly with the measures of 1867.

The occasional observations I have made of this object during nine years do not appear to me to afford any evidence of a permanent change of the lunar surface at this spot. The different aspects which this object assumes under different conditions of illumination appear to me to be exactly the same as those which it presented in 1867.

The measures in 1867 were made with a refractor of 8 inches aperture and an eye-piece magnifying 500 diameters. The observations were made in 1873, with a refractor of 15 inches aperture and an eye-piece giving a power of 420 diameters.

I append a few extracts from my note-book of some of the appearances which this object presents :—

1866, December 14.—With Savart's polariscope. The bands passed unbroken across *Linné*, which appeared as a white spot.

1867, July 8, $7^h\ 10^m$.—As a shallow oval crater, a little shadow thrown by the western margin.

Ditto, ditto, 8^h to $8^h\ 30^m$.—Small cone as a little hill a little west of centre. Shadow cast by it pointed, and extending nearly to the eastern margin.

July 9, $8^h\ 30^m$ to $9^h\ 30^m$.—The boundary of white spot gradually passes into surrounding surface, more irregular at some points. Measures will differ as more or less of the faint outline is included, and according to the points of the outline selected.

November 6.—Several outlying portions of white surface of a smaller degree of brilliancy in contiguity, if not physically connected, with *Linné*. The boundary especially uncertain from this cause at the western end and also along the northern side. The southern side is less interrupted, and here the bright surface forming *Linné* terminates more abruptly. There are other spots on the Moon which possess a similar want of distinctness.

November 11 (One day short of full Moon).—Under this illumination the small crater, at least its rim, is much brighter than the flat crater on which it occurs. The peculiarity of its appearance at this time consists in the rapid diminution of brightness from the centre outwards.

December 4.—An offshoot of bright surface from the bright spot towards the S.W. was very distinct.

1868, March 30.—Nearly as sharply defined as neighbouring craters, not the usual cloudy appearance. Ring wall high, especially on the western side. Curious effect of eye retaining a defined object. The small crater appears, by motion of air, to swing to and fro over the white patch.

1873, July 4, 8^h to 9^h 30^m.—*Linné* quite nebulous at edges, no regular outline, but broken up by small projections. The oval appeared in the direction of E.S.E. to W.N.W.

August 1.—The direction of the longer axis seems to change while I am looking, sometimes E.S.E. to W.N.W., at others from S.W. to N.E. (the true direction of the shallow crater); the former direction (at right angles to the true one) appears more persistent.

December 27, 6^h 30^m to 7^h.—At first oval crater very distinct. Half an hour later I lost the outline of this in a bright nebulous patch. This nebulous light more marked on the longer sides of the oval, so that, as on former occasions, the oval form of the patch seemed sometimes in one direction and sometimes in the opposite direction. The small crater seemed to vary in position on the white patch, as in former years, probably from the varying visibility (from atmospheric changes) of the outside portions of the large crater.

On the Colour and Brightness of Stars as measured with a new Photometer. By W. H. M. Christie, Esq., M.A., Fellow of Trinity College, Cambridge.

Various plans have been proposed for referring the colours of stars to a standard scale, but most of them depend on the comparison of the star's light with painted surfaces, which, being necessarily viewed by artificial light, are liable to play the observer false tricks, especially where any colour containing green is concerned. Besides this, the eye is so readily biassed by any coloured light previously seen, that no reliable results are to be expected, unless the star and the comparison light are placed side by side. This is effected in Zöllner's photometer; but the comparison of an artificial star with the interference image of a star as seen in a telescope can hardly be considered satisfactory, at any rate in determining colour, and the use of the tints of polarization is open to the two-fold objection, that it must be difficult to match any given *hue*, and nearly impracticable to express the result in terms of the

three primary colours. As every hue in nature is compounded of white and some colour of the spectrum, or which is the same thing, of the three primary colours, any given hue must be a function of two quantities, and therefore cannot be expressed in terms of a single circle reading, as Zöllner appears to have attempted. In fact, we may express both the intensity and hue of any object by the quantities of red, green, and blue in it, or, if we prefer it, by the quantity of white, together with the quantity and position of some colour in the spectrum.

The instrument which I am about to describe is on the principle of Clerk-Maxwell's Colour-box (described in the *Philosophical Transactions*, 1860), in which definite quantities of red, green, and blue light can be compounded so as to match any given hue.

The objects which I have sought to attain are :

1. To place two uniform surfaces of sensible area side by side and in contact for comparison.
2. To determine both the intensity and hue of the light coming from a star or other body.
3. To obtain a fixed standard of whiteness.
4. To compare the colours of different portions of the disks of the Moon and planets.

It being premised that the photometer is attached to the eye-end of an equatoreal by means of a ring clamp, the general arrangement is as follows :—

The beam of parallel rays, of which the light of a star consists, after passing through the object-glass and eye-piece of the equatoreal, falls on a lens L, at the principal focus of which the eye is placed, so that a round spot of light is seen on one half of the lens, whilst the other half receives the compound comparison light, which will, if a suitable diaphragm be placed in front of the lens, be seen as a round spot in contact with that formed by the star, and exactly of the same size. The diameter of the spot is of course the ratio of the aperture to the magnifying power. The compound light falls on L after having passed through the collimating lens C and prisms P, from three slits, R, G, B, opposite which are the three flames of a paraffin lamp M, of peculiar construction.* The slits R, G, B are so adjusted, that of the light coming from R, and forming a spectrum at the eye-end, the slit E (behind which the eye is placed) will receive red

* This arrangement, in fact, constitutes a spectroscope in which three spectra are superposed at the eye-end, but it must be remembered that no eye-piece is used, and that a coloured field, not a spectrum, is seen by the eye at E. The dispersion is given by two flint prisms of 60° (forming a spectrum 4·5 inches in length from A to H at the lamp-end), but for convenience the spectrum is turned through 90° by the use of a right-angled reflecting prism. In any photometric enquiry, simple prisms are much to be preferred to any form of direct vision prism, as with the former suitable stops can be placed to cut off false spectra. The focal lengths of the lenses C and L are 14 and 6 inches respectively.

rays of a certain refrangibility, whilst it will receive green rays from G, and blue rays from B; the resultant light will therefore be a compound of red, green, and blue, the quantities of each being measured by the products of the width of the slit E into the areas of the slits R, G, and B, respectively.

The first thing, then, is to have the means of varying these areas without removing the eye from E. Without entering into detail, it may be sufficient to explain that the slits slide on a dovetail, and are closed by spiral springs, whilst they are opened by the rods D, E, F, which terminate in a screw and a taper conical head. As the rod is turned, the cone is driven between the jaws of the slit, which thus opens equally on both sides, so that the colour of the light remains unaltered. A horizontal slit, which can, before the observation, be adjusted to any convenient width (not necessarily the same for the three colours) is placed in front of all; the width of the eye-slit is not usually altered unless for stars widely differing in brightness. Having carefully matched the light of any given star by turning the milled heads D, E, F, we have next to measure the areas of the slits R, G, B as accurately as possible. A convenient way of doing this is, after removing the lamp, to place an ordinary scale dynamometer in the channels H, I, K successively, these being so adjusted that the light from a gas jet reflected up the collimator by a tin reflector placed outside C will enter the dynamometer after passing through the slit; both the width and height of the slit can thus be read off by estimation to $\frac{1}{3000}$ inch, the product of the numbers giving the area required. I have also used a graduated steel wedge, but prefer the dynamometer, as not likely to disturb the slits in any way.

The lamp M, on which everything depends, has three ordinary paraffin burners soldered side by side, so as to be opposite the three slits; the chimney is rectangular and of tin, with two narrow horizontal openings covered with glass, one in front to illumine the slits, the other behind to verify the height of the flames at any time.* Owing to the strong draught necessary in a paraffin lamp, it occurred to me that the lamp might be considerably inclined without affecting its burning, and such I have found to be the case; the flame goes straight up the chimney, and remains free from smoke even when the lamp is inclined 60° or 70°, so that by fixing it at an angle of 25° or 30° to the plane of the slits we can work up to the zenith. That objectionable article, a swinging lamp, is thus got rid of; and though there may be a little difficulty in managing three flames, they are, I am convinced, less troublesome than any arrangement for reflecting the light from a single flame down the collimator. The performance of the lamp is very satisfactory, and its light remarkably constant and pure.

* This can also be done conveniently by viewing the light from each slit in succession through an eye-piece at E, where it will be easy to see when the white part of the flame falls on the slit.

I will now pass to the third point, a fixed standard; and it will be necessary to discuss this question at some length, though I do not propose in the present paper to enter on the subject of colour in general, but only in its bearing on the constitution of the heavenly bodies. My first object being to measure the light of stars, I have adopted provisionally the positions of the three standard colours which Clerk-Maxwell gives in the *Philosophical Transactions*, 1860, viz.: Red, wave length (in Fraunhofer's measure) 2328; Green, wave length 1914; and Blue, wave length 1717. Now it appears from Clerk-Maxwell's observations that any colour of the spectrum can be matched by a mixture of any two of these three, but that to match white light we require a mixture of all three, in certain definite proportions. It becomes then a matter of some interest to settle what these proportions are. White light may be taken as that which contains an equal admixture of all the rays of the spectrum, or in other words, which has the same energy for equal areas in all parts of the diffraction spectrum, but in the present state of our knowledge of the relative intensities of the colour sensations this definition has no practical value, and we must be content with assuming some convenient standard, all that is necessary being that it should agree tolerably well with the ordinary notions of white light, and should be universally accessible and not liable to change.

There must evidently be something arbitrary in this selection; but provided our measures are all reduced on one system, it will be easy afterwards to refer them to the absolute standard, should it afterwards be found practicable to determine it.

The only method of eliminating all instrumental influences (such as absorption in the object-glass and eye-piece), is clearly to choose some heavenly body which will be affected in the same degree by these sources of error. The Moon would be the most natural standard, but here we are met by the objection of possible change and the variety of tint introduces a practical difficulty, though by selecting such an object as *Aristarchus* we may get at any rate a provisional standard to be afterwards compared with the ultimate standard. For the latter I would suggest the planet *Venus*, though I have not obtained any measures yet, owing to the present unfavourable position of this planet.

The average of a number of stars would answer the same purpose, and might further serve as the base of a scale of magnitudes, with this advantage over Sir J. Herschel's proposal of *a Centauri* (which, by the way, is not universally accessible), that it is in the highest degree improbable that the average of a number of stars in all parts of the heavens should change appreciably. Considering that observers were more likely to agree in their estimate of 6th magnitude stars, I have selected a number of such stars in which Piazzi, Argelander and Heis agree, thereby giving a great security against change: these should be used as clock stars are in determination of right ascensions, several of

them being observed in connection with other objects. Taking the mean of these as a standard for 6-magnitude, and Pogson's value of the Light ratio R=2·512, we should have a consistent scale, agreeing well on the whole with Sir W. Herschel's determination of the first six magnitudes, and also with the scale adopted by modern observers. The following table expresses the light of a star of any magnitude in terms of that of a 6-magnitude star :—

Magnitude	1	2	3	4	5	6	7	8
Light ratio	100·000	39·810	15·850	6·310	2·512	1·000	0·398	0·159

Since 5 Log R=2 the values for the lower magnitudes are given by shifting the decimal point two places in the value for the magnitude which is 5 higher in the scale.

But to express the light of a star, as measured with the photometer, in this scale of magnitudes it is necessary to have more observations of the relative intensities of the red, green, and blue, than I have yet been able to obtain. From a mean of 7 comparisons of these in succession with a constant white light (paraffin) it appears that in my instrument 5·9 parts of Red are equivalent in brightness to 14·6 parts of Green, or to 100 parts of Blue. These are expressed in terms of the breadth of the slits in the dispersion spectrum, where the red rays are crowded together and the blue spread out so that the unit is different for rays of different refrangibility; in order to refer them to the diffraction spectrum we must lay down the curve of which the abscissæ are scale readings of the spectroscope and the ordinates wave lengths, the tangent of the inclination of the tangent at any point of this curve will then be the factor for converting the breadth of the slit at that point. For the three standard colours, the factors are 1·84, 0·86, and 0·48, in my instrument. The intensities of the Red, Green, and Blue, in the diffraction spectrum would thus be roughly as 1 : 1 : 4.

It remains to explain how the positions of the principal Fraunhofer lines are determined. An eye-piece being applied at the eye-end, and the eye-slit (which is on a slide) placed in a definite position near the line of collimation, one of the slits at the lamp-end is moved till the line to be measured is brought into the centre of the slit; the scale at the lamp-end (which is divided to $\frac{1}{50}$ in.) is then read off. In practice, it has been found convenient to set the three slits to definite positions, and to move the eye-slit till the sodium lines in the light of the red slit appear, and then, by means of a scale at the eye-end, to set the eye-slit to its proper position.

It might seem that by comparing two similar surfaces, personality would be entirely eliminated, but this is not the case if there be anything like selective absorption in the eye of the observer. In his able paper, previously cited, Clerk-Maxwell points out that there is in most eyes in a greater or less degree

such an absorption of the rays between the Fraunhofer lines E and F, but confined apparently to the yellow spot of Sœmmering.

With regard to the isolation of a portion of the Moon's or a planet's disk, it may be remarked that, as an image (which may be viewed by an eye-piece) is formed at the eye-slit, the latter may be made use of for this purpose; I have, however, preferred the use of a pinhole at the principal focus of the equatoreal, as enabling the observer to examine a smaller portion of the surface; the diameter of my smallest pinhole is 6″, and it would not be difficult to make one of half that size. For double stars, however, the former method has generally been used.

Though the instrument (which was made for me by Mr. Browning) was received at the end of September 1872, I have not yet been able to accumulate sufficient observations for any certain conclusions, partly owing to the experiments necessary in an inquiry of this kind, but, in a far greater degree, to the very unfavourable weather of the past year. As it is, I have felt constrained to make use of some nights which were hardly suitable for delicate observations. The photometer was attached to the Great Equatoreal of the Royal Observatory, Greenwich, and the subjoined observations were made with it in the last few months; the earlier measures are reserved for future discussion.

Date, 1873.	Object.	Z.D.	No. of Obs.	Areas of Slits. Red.	Green.	Blue.	Relative Intensities. Red.	Green.	Blue.	Apparent Brightness.	Corrected Brightness.
		°									
Aug. 11	☽ Aristarchus	60	1	676	2960	5110	1150	2030	511	3690	4610
Sept. 26	95 Herculis B	55	1	34·6	140	470	58·8	95·9	47·0	202	236
,,	,, A	58	1	33·1	140	373	56·3	95·9	37·3	190	230
,,	β' Lyræ	49	1	83·2	485	1010	141	332	101	574	631
,,	β Cygni A	52	1	132	481	46·8	225	329	4·7	559	632
,,	,, B	58	1	22·9	55·8	173	38·9	38·2	17·3	94·4	114
,,	η Pegasi	38	2	126	756	857	214	518	85·7	818	851
Sept. 27	95 Herculis B	53	2	15·9	99·2	64·3	27·0	68·0	6·4	101	115
,,	,, A	55	2	15·9	118	211	27·0	80·8	21·1	129	151
,,	β' Lyræ	47	4	59·0	445	660	100	305	66·0	471	508
,,	γ Lyræ	52	2	64·3	551	834	109	377	83·4	569	643
,,	χ^1 Cygni	57	2	14·7	105	117	25·0	71·9	11·7	109	130
,,	χ^2 Cygni	64	2	23·2	87·5	9·4	39·4	59·9	0·9	100	135
Oct. 8	β' Lyræ	46	2	42·8	256	484	72·8	175	48·4	296	317
,,	γ Lyræ	45	2	59·9	410	739	102	281	73·9	457	489
Oct. 16	β' Lyræ	48	2	75·6	704	828	129	482	82·8	694	763
,,	γ Lyræ	55	1	82·0	767	899	139	525	89·9	754	882
,,	η Pegasi	42	2	137	657	322	233	450	32·2	715	751
Oct. 29	β' Lyræ	44	2	94·0	481	325	160	330	32·5	523	554
,,	γ Lyræ	47	3	89·0	557	495	151	382	49·5	583	629

Date 1873.	Object.	Z.D.	No. of Obs.	Areas of Slits. Red.	Green.	Blue.	Relative Intensities. Red.	Green.	Blue.	Apparent Bright-ness.	Corrected Bright-ness.
		°									
Nov. 1	Schjell. 241	55	2	9·1	46·2	133	15·5	31·6	13·3	60·4	70·7
„	β' Lyræ	61	3	60·0	661	655	102	453	65·5	621	789
Nov. 3	β' Lyræ	53	3	69·4	491	441	118	336	44·1	498	568
„	γ Lyræ	58	2	91·1	656	762	155	449	76·2	680	823
„	Schjell. 241	69	2	13·8	43·8	0·6	23·5	30·0	0·1	53·6	80·9
„	41 Pegasi *	49	1	9·0	43·5	56·4	15·3	29·8	5·6	50·7	55·8
„	40 Pegasi *	51	1	15·3	66·1	2·4	26·0	45·3	0·2	71·5	80·1
„	45 Pegasi *	52	1	14·4	60·3	0·0	24·5	41·3	0·0	65·8	74·4
„	52 Pegasi *	58	1	9·5	63·8	0·0	16·1	43·7	0·0	59·8	72·4
„	η Pegasi	53	2	238	1500	1470	405	1030	147	1580	1800
„	β Pegasi	54	1	290	1910	808	493	1310	80·8	1880	2160
„	☽ Mare Imbrium	45	1	205	1770	3470	349	1210	347	1910	2040
Nov. 12	Schjell. 241	64	3	8·1	32·4	0·0	13·8	22·2	0·0	36·0	48·6
„	40 Pegasi *	44	1	10·4	51·8	0·0	17·7	35·5	0·0	53·2	56·4
„	41 Pegasi *	46	1	5·7	33·8	78·4	9·7	23·2	7·8	40·7	43·5
„	η Pegasi	44	2	129	1150	468	219	788	46·8	1050	1110
„	β Pegasi	50	2	278	1320	26	473	904	2·6	1380	1530
„	60 Pegasi *	59	1	6·5	39·5	14·0	11·1	27·1	1·4	39.6	48·7
„	n^1 Orionis *	52	1	7·5	130	98·0	12·8	89·1	9·8	112	127
„	n^2 Orionis *	51	1	9·0	129	14·0	15·3	88·4	1·4	105	118
Nov. 30	η Pegasi	53	1	97·5	623	268	166	427	26·8	620	707
„	β Pegasi	60	1	166	847	258	282	580	25·8	888	1110
„	λ Tauri	42	2	65·3	425	466	111	291	46·6	449	471
„	n^1 Orionis *	50	1	13·6	101	112	23·1	69·2	11·2	104	114
„	n^2 Orionis *	50	1	12·8	101	32·7	21·8	69·2	3·3	94·3	105
Dec. 31	η Pegasi	63	3	117	710	293	199	486	29·3	714	942
„	λ Tauri	47	2	94·4	497	597	160	340	59·7	560	599
„	n^1 Orionis *	53	2	9·8	80·8	146	16·7	55·3	14·6	86·6	98·7

* Standard 6th magnitude stars:

Aug. 11. Hazy clouds; brightness of moon variable; many attempts made to get a good match. Diameter of Pinhole, 8″·6.
Sept. 26. East wind; images bad, and brightness variable.
Sept. 27. Definition good at times; south wind.
Oct. 8. Sky hazy, and definition variable. Bright moonlight.
Oct. 29. Rather foggy; images not good. Moonlight.
Nov. 3. Images fairly good. Diameter of Pinhole (for Moon), 8″·6.
Nov. 12. East wind; images bad, generally granulated, with rare fits of fair definition; colours of stars seemed variable.
Nov. 30. Sky hazy, except for n^1 and n^2 *Orionis*. Moonlight.
Dec. 31. Brightness and colour of stars very variable.

The numbers given in the columns headed Areas of Slits express the areas of the several slits in square $\frac{1}{500}$ inch, all the measures being referred to the same width of eye-slit ($\frac{1}{50}$ inch), and three significant figures only being retained ; the same power (130) was used throughout on the equatoreal, though with very bright stars there is an advantage in using a lower power, whilst for stars below 6 mag. a high power which concentrates their light is perhaps better.

The next three columns are formed by multiplying the three preceding columns by $\frac{10}{5{\cdot}9}=1{\cdot}70$, $\frac{10}{14{\cdot}6}={\cdot}685$, and $\frac{10}{100}=0{\cdot}1$, and express the intensities of the three colours in each case ; the sum of these numbers is taken in the next column, which gives the apparent brightness subject to a correction for atmospheric absorption. This correction (taken from Seidel's Table) is applied in the last column, which therefore expresses the brightness of a star as it would be seen at the zenith. There is very little doubt that a different correction ought to be applied for each colour, so that the present reduction must be regarded as provisional, but it will serve to illustrate the method which may be advantageously employed. I have not thought it worth while at present to reduce these observations to the scale of magnitudes, which I have proposed. This will be done more advantageously when more observations have been collected, for it is evident that even with the selected 6th magnitude stars, there is no such approach to equality as we have a right to demand. There can be no doubt, for instance, that both n^1 and n^2 *Orionis* are now nearly twice as bright as an average 6-magnitude star, and yet we find Piazzi, Argelander, and Heis agreeing to one-sixth of a magnitude, and all three calling them 6·0 magnitude.

As might be expected, in observations of such a nature with a new instrument, there are certain anomalies in these measures which are probably due in part to scintillation, the effect of which is to make the star's light change from orange or yellow to blue-purple ; this was remarked very distinctly on December 31. Another cause of error which, however, affects the brightness rather than the colour of stars, is the dryness of the air : with an east wind the spot of light usually becomes granulated, and there is in consequence a strong tendency to over-estimate its brightness. From a comparison of the discordance of the separate results for the same star on any one night, the mean error of a single measure appears to be about 10 per cent., equivalent to 0·1 of a magnitude. But after due allowance has been made for all probable sources of error, there remain discordances which seem to imply rapid changes of colour in some of the stars examined. Now change of colour shows a physical change, for it

is extremely improbable that such a phenomenon should be caused by rotation. We know that as the temperature of a solid is raised, the colour changes from red to white, radiations of all wave lengths increasing, but the blue more than the red; now, we see something similar to this in the results for *η Pegasi*, *β Pegasi*, and Schjellerup 241; but I do not wish to insist strongly on the few measures I have obtained as yet, though there can be no doubt that *η Pegasi* is a new variable star of moderate period. Owing to unfavourable weather, the observations are for the most part separated by such long intervals as to give but little information on the light curves, and from the same cause I have been baffled in my attempts at observing a minimum of *β Lyræ* or *Algol*; the observation of λ *Tauri* on November 30, at $12^h\ 25^m$ G. M. T., was close to the minimum. If changes in the colours of stars are due to gases, as would seem probable, the law of radiation, given above for solids, will have to be modified; thus, an increase of temperature (in the case of hydrogen) would, I fancy, increase the red and blue without affecting the green. But to get further information on this point, we require measures of the colours of gaseous bodies under various conditions of temperature and pressure. In this way the photometer will supplement the spectroscope, for though it cannot give us the same definite information on the constituent elements of the heavenly bodies, it may tell us something about changes of temperature and pressure, and may be readily applied to the fainter orders of stars, a field left untouched by the spectroscope. Even with the brighter stars, much may be done by the help of a good photometer with which the *absolute* amounts of the three standard colours can be measured, for among the numerous lines of stellar spectra it must be very difficult to trace changes, but a change in colour will at any rate guide the search; thus, if the red and green increase together, a bright line has probably appeared in the intermediate yellow, or a dark line has faded out; again, if the green and blue decrease together, a bright line has probably faded out in the region near F.

A few words, in conclusion, on another application of this instrument. Recent researches have shown the importance of determining the changes in the relative brightness of the lines in the spectra of gases with changes of temperature; now, though I have not tried the experiment, yet I conceive there would be no difficulty in combining the photometer with a spectroscope so as to match the different bright lines, which might be done either by placing the lens L at the principal focus of the spectroscope, or by the use of an eye-piece as in measures of stars.

And now, having pointed out the results which we may hope for from such measures, I would earnestly commend this branch of astronomy to all those who have equatoreals, and do not know exactly what useful work to take up with them. With a very moderate expenditure of time and money they may do valuable work in sidereal photometry with the promise of a rich harvest of discoveries.

My object in this paper has been not so much to give results as to explain the method of using such an instrument, and, if possible, to induce others to take up a most interesting inquiry; this must be my apology for having, in some cases, drawn conclusions from very insufficient data, instead of waiting till further materials were accumulated, though I trust that such statements as I have ventured on will be regarded merely as working hypotheses intended to suggest new lines of research.

Blackheath,
1874, *January* 8.

Note on an Ancient Astrolabe. By R. J. Lecky, Esq.

This Astrolabe was found about the year 1846, under a large flat stone, on the mountain side opposite the northern entrance of Valencia Harbour, County of Kerry, Ireland. It was in a leather case, which went to pieces on being handled. Bewicke Blackburn, Esq. C.E., who was then a resident on the Island of Valencia, purchased it, and gave it to me a short time afterwards.

About six months after this, I was again at Valencia, and accompanied by Mr. Blackburn, took the lad who had found it to the place; and, on searching amongst the rocks, found a piece of the old case, thus confirming the lad's report.

The instrument is of brass, weighs 5lbs. 9¾ oz. and is 7 inches diameter by over ⅝ths of an inch in thickness; with a strong outer circle, the lower part being cast solid in order to give it weight, and at the upper part is a double or cross joint, with a ring for suspension. The vanes are 1¼ inch square, with very minute sight-holes just large enough to admit a very fine needle; they are cast solid on to the cross arm, which is connected to the body of the instrument by a turned pin with a neatly cut rose head and ornamental forelock: this arm ends with a pointer at each end, the fiducial edges of which range with the sight-holes. The upper half of the circle is alone divided into degrees; and these are not numbered, neither is there a name or date on the instrument. It has evidently been left unfinished; but all the work, as far as it has gone, has been carefully executed: the double joint, cross arm centre pin, and vanes have been thoroughly well made and fitted. It is impossible now to say where this Astrolabe was made, or where it may have originally come from; but, judging from the age in which they were used, and the style of the workmanship, the idea that it belonged to the Spanish Armada seems probable, and that it may have been stolen from the wreck of one of these vessels which occurred on the Blasquet Islands within view of the place were it was found.

Observations of Occultations of Stars by the Moon, and of Phenomena of Jupiter's Satellites, made at the Royal Observatory, Greenwich, in the Year 1873.

(*Communicated by the Astronomer Royal.*)

Occultations of Stars by the Moon.

Day of Obs. 1873.	Phenomenon.	Telescope.*	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
May 1	Disapp. of 39 Geminorum	G. Eq.	Dark	8 22 6·8	C
,,	,, Arg. +26°. No. 1410	,,	,,	8 44 21·8	,,
July 4	,, λ Virginis	Altaz.	,,	10 15 47·0	E
Aug. 9	,, τ^2 Aquarii	G. Eq.	Bright	10 5 5·4	G
Oct. 3	,, τ^2 Aquarii	Altaz.	Dark	7 47 31·5	E
Oct. 13	Reapp. of λ Cancri	E. Eq.	,,	13 0 53·9	C
Dec. 1	Disapp. of *o* Arietis	,,	,,	12 42 54·4	,,
Dec. 24	,, τ^2 Aquarii	Altaz.	,,	4 31 52·7	A D
,,	Reapp. of τ^2 Aquarii	,,	Bright	5 28 51·5	,,
,,	,, τ^2 Aquarii	E. Eq.	,,	5 28 49·8	E

Phenomena of Jupiter's Satellites.

Day of Obs. 1873.	Satellite.	Phenomenon.	Telescope.	Mean Solar Time of Observation. h m s	Mean Solar Time from N. A. h m s	Observer.
Jan. 10	I. (*a*)	Ecl. disapp.	G. Eq.	10 16 4·6	10 15 43·1	H C
,,	II. (*b*)	,,	,,	10 20 39·8	10 20 3·1	,,
,,	IV. (*c*)	Ecl. reapp.	,,	10 54 22·3	10 54 29·2	,,
,,	IV.	Occ. dis. first cont.	,,	13 33 59·1	13 43	,,
,,	IV. (*d*)	,, bisec.	,,	13 40 28·0		
Jan. 26	I. (*e*)	Ecl. disapp.	,,	8 31 11·0	8 30 45·8	W C
Mar. 8	II. (*f*)	,, reapp.	,,	10 2 28·7	10 2 53·8	L
Mar. 13	I.	,, ,,	,,	11 5 48·8	11 6 16·5	E
Mar. 14	III. (*g*)	,, ,,	,,	7 5 42·7	7 5 20·9	A D
,,	I.	Tr. egr. first cont.	,,	7 45 36·2		
,,	I. (*h*)	,, bisec.	,,	7 47 35·9	7 50	H C
,,	I.	,, last cont.	,,	7 49 20·6		
Mar. 22	I. (*i*)	Ecl. reapp.	,,	7 29 8·6	7 29 22·0	A D
Mar. 26	IV. (*k*)	Tr. egr. bisec.	E. Eq.	10 1 20·2	10 7	C
,,	IV.	,, last cont.	,,	10 5 19·6		

* The clear aperture of the object-glass of the Great Equatoreal is $12\frac{3}{4}$ inches, of the East Equatoreal 6·7 inches, and of the Altazimuth $3\frac{3}{4}$ inches.

Day of Obs. 1873.	Satellite	Phenomenon.	Telescope.	Mean Solar Time of Observation. h m s	Mean Solar Time from N.A. h m s	Observer.
Mar. 27	I. (*l*)	Ecl. disapp.	G. Eq.	14 55 7·5	14 55 17·8	A D
Mar. 28	III.	Occ. dis. first cont.	,,	7 43 52·8		
,,	III. (*m*)	,, bisec.	,,	7 48 22·1	7 46	W C
,,	III.	,, last cont.	,,	7 52 11·4		
,,	III. (*n*)	,, first cont.	E. Eq.	7 47 9·2		
,,	III.	,, last cont.	,,	7 51 38·4	7 46	C
,,	I.	Tr. egr. first cont.	G. Eq.	11 20 46·7		
,,	I.	,, bisec.	,,	11 22 1·5	11 23	,,
,,	I.	,, last cont.	,,	11 23 46·2		
,,	III.	Occ. reapp. last cont.	,,	11 25 15·9	11 28	,,
,,	III. (*o*)	Ecl. disapp.	,,	11 34 34·4	11 32 1·6	,,
Apr. 9	II. (*p*)	,, reapp.	,,	9 47 9·9	9 47 20·8	H C
Apr. 21	I.	,, ,,	E. Eq.	9 37 6·6	9 37 4·9	C
Apr. 29	I.	Tr. egr. first cont.	G. Eq.	7 37 43·5		
,,	I. (*q*)	,, bisec.	,,	7 39 28·2	7 41	H C
,,	I.	,, last cont.	,,	7 41 57·8		
May 24	IV. (*r*)	Ecl. reapp.	,,	10 48 20·4	10 48 10·4	L

Notes.

(*a*) The first sensible diminution of brightness was noticed $1^m\ 30^s$ before the time recorded above.

(*b*) A diminution of brightness was noticed $2^m\ 30^s$ before the recorded time.

(*c*) The observation was not very satisfactory, owing to cloud; but the observer considers that the time noted is not more than a few seconds late.

(*d*) The satellite had not totally disappeared three minutes after the time recorded at this observation, but *Jupiter* became so tremulous that it was impossible to estimate the time of last contact. Magnifying power used = 310.

(*e*) Bad definition. Power = 285.

(*f*) The satellite was at its full brilliancy about three minutes after the time recorded above. Power = 220.

(*g*) The satellite did not attain its full brightness till six or seven minutes after the recorded time. Power = 310.

(*h*) Observation good; the satellite was distinctly visible on the planet's disk at first contact. Power = 220.

(*i*) The satellite had attained its full brightness three minutes after the time recorded above; both the planet and satellite were very tremulous. Power = 500.

(*k*) Images very bad. Power = 130.

(*l*) The satellite was at its full brightness $2^m\ 37^s$ after the time recorded above; the images were not good, the sky being rather misty. Power = 220.

(*m*) Definition very good, though light passing clouds occasionally made the planet very faint. The observed time of last contact is considered fairly accurate. Power = 310.

(*n*) Satellite faint; sky very thick.

(*o*) The first diminution of light was noticed $2^m\ 35^s$ before the time recorded above.

(*p*) Observation good; the satellite attained its full brightness about three minutes after the time recorded above. Power = 310.

(*q*) Observation good; the satellite at first contact was distinctly seen on the disk of *Jupiter*. Power = 220.

(*r*) The satellite attained its full brilliancy $4^m\ 58^s$ after the time recorded above. Power = 60.

The initials W C, E, C, L, A D, H C, and G, are those of Messrs. Christie, Ellis, Criswick, Lynn, Downing, Carpenter, and Goldney.

Note on a Paper which appeared in the last Supplementary Number of the Monthly Notices (page 533). By Richd. A. Proctor, Esq.

I so far depart from a determination announced last summer, and sufficiently indicated in my last contributions to the *Monthly Notices* of the Royal Astronomical Society, as to send the present Note, the object of which is to express my regret for the circumstances which led to the appearance in the *Monthly Notices* of a paper "On the Antarctic and sub-Antarctic Regions suitable for observing the Transit of 1874." If I had known earlier of the arrangements described by Sir George B. Airy, at the November meeting of the Society, my paper would not have appeared, since in fact it could have served no useful purpose. Those arrangements seem to me to meet the requirements of the occasion as well as can, at the present time, be expected.

I take this opportunity of making three remarks relating to matters mentioned at the November meeting of the Society:—

First, I have never questioned in the slightest degree the energy with which the Astronomer Royal for England is superintending the details of the proposed expeditions.

Secondly, I have not urged as desirable the occupation of any specified stations in Antarctic or sub-Antarctic seas; my suggestions (the justice of which I remain convinced of) have related to the *search* for such stations in due time. These suggestions I regard as calculated, *when they were made*, to be extremely useful.

Thirdly, the above-mentioned essay in the *Monthly Notices* of the Society was accompanied by a foot-note clearly indicating that the paper was inserted on my responsibility alone.

Westminster Hotel, New York,
1873, *December* 26.

Note on the suspected Change of Position and Magnitude of the Stars between ε and 5 Lyræ. By E. Dunkin, Esq.

It was mentioned by Capt. Noble, at the January meeting, that Mr. Prince had been examining a drawing of this group made by the Rev. W. R. Dawes many years ago, and that he had discovered that it agreed generally with that which was given in the last number of the *Monthly Notices* as the appearance of the group at the present time. Mr. Lassell also explained to the Meeting that a drawing made by him many years ago represents the components of the group in the same relative position and magnitude as in that sketched by Mr. Prince. The evidence thus brought forward clearly proves that no sensible change has taken place either in the position or magnitude of the intermediate stars, and that the representation of the group given in the *Celestial Cycle* is inaccurate. Unfortunately, Admiral Smyth made no separate measurements of the three intermediate stars in question, and it is very probable that they were inserted in his diagram without much attention to their accurate position. Dr. De La Rue and others also remarked at the meeting, that a comparison with the diagrams in the *Celestial Cycle* should never be relied on as evidence of change, and it is therefore necessary, when comparing observations with those in the *Cycle*, to depend alone upon the measures given in the text.

Kenwyn, Kidbrooke Park Road, Blackheath,
1874, *January* 10.

Observed Position of ξ Ursæ Majoris in December 1873.
By Wentworth Erck, Esq.

Epoch.	Position.	Mean.
	° 341·15	
1873.936	1·39	341°·68
	2·50	

Each of the above is the mean of 10 measures:

Mean sidereal time of observation = $10^h\ 20^m$, when the components were nearly in the vertical:

As to the distance; clear space equal one-third diameter of apparent disk as observed with $7\frac{1}{2}$-inch aperture:

The components were in contact with 6-inch aperture. Hence the distance was estimated at $0''\cdot8$.

As a comparison star I measured γ *Arietis*, said by Dawes to be unchanged since 1830. I found the position thereof to be 358°·28.

Telescope, the $7\frac{1}{2}$-inch Alvan Clark: Power 750.

Sherrington, Bray,
1873, *December* 19.

Observations of Double Stars. By C. E. Burton, Esq.

No.	Name of Star.	Position. °	Distance. ″		Epoch.	No. of Measures.
1.	Castor	237·8	5·25		1873·981	15
2.	ζ Cancri	143·3	0·6	estimated	1873·956	10
3.	ξ Ursæ Majoris	342·4	0·8	„	1873·945	10
4.	32 Orionis	189·8	0·5	„	1873·967	5

1. The distance is the result of two measures on each side of the zero, differing by 0″·1. Observed on two nights with powers of 400 and 940.
2. Observed on two nights, five measures with power 400 and five with 940.
3. Power 400.
4. Measures extremely difficult; power 940. This star is evidently a binary.

The black spaces between the disks of the last three doubles were carefully estimated as follow:—

ζ Cancri, with power 400, one-half apparent diameter of disk of either star.
„ „ 940, barely equal to apparent diameter of disk of A.
ξ Ursæ Majoris „ 400, apparent diameter of disk of either star.
32 Orionis „ 940, apparent diameter of disk of B.

The observations were made with a 12-inch Newtonian equatoreal and position-wire micrometer.

Loughlinstown, Co. Dublin,
1874, *January* 7.

Two Occultations of τ^2 *Aquarii observed during* 1873.
By Captain Noble.

The first of these occultations occurred on August 9, 1873, when the Moon was very low down.

At the time of disappearance, which took place at the bright limb, the bubbling and boiling were so tremendous that when the star was nearly in contact with the lunar limb, the latter seemed actually to involve and overlap it by its twists and twirls. Nevertheless, the actual disappearance was fairly observed at $19^h\ 18^m\ 32^s$ L. S. T. $= 10^h\ 4^m\ 41^s{\cdot}2$ L. M. T.

The re-appearance took place, of course, at the dark limb; but the Moon being only 1·8 day past the full, pretty near to the illuminated part of her western limb. It was quite sharp, and happened at $20^h\ 19^m\ 6^s{\cdot}5$ L. S. T. $= 11^h\ 5^m\ 5^s{\cdot}8$ L. M. T. The orange tint of the star was very perceptible on its emersion. Power 255, on my 4·2-inch Ross equatoreal, adjusted on a star.

The second occultation was that of December 24.

The star disappeared instantaneously at the Moon's dark limb at $22^h\ 45^m\ 24^s{\cdot}1$ L. S. T. $= 4^h\ 32^m\ 19^s{\cdot}8$ L. M. T.

Its re-appearance was scarcely instantaneous; rather resembling the rising of a tiny planet from behind the bright limb of the Moon. It happened at $23^h\ 41^m\ 56^s$ L. S. T. $= 5^h\ 29^m\ 36^s{\cdot}5$ L. M. T. Power 255, adjusted on the star.

Forest Lodge, Maresfield, Sussex.

Observations of Faye's Comet (VI., 1873) *at the Observatory at Marseilles.*

This Comet appears to have been observed only at Marseilles by M. Stéphan. On each day of observation it was extremely faint. An observation made on September 3 is inserted on page 42, vol. xxxiv. of the *Monthly Notices*, and two others made on November 28 and 30 give the following places:—

1873.	Time of Observation, Marseilles M. T. h	m	s	Observed R. A. h	m	s	Observed N. P. D. °	′	″	Comparison Star.
Nov. 28	16	24	55	9	16	19·75	89	37	25·6	*a*
Nov. 30	17	11	2	9	17	13·34	89	54	37·8	*b*

The corrections to the ephemeris in the *Berliner Jahrbuch* are:

	R. A. s	N. P. D. ″
Nov. 28	+0·73	−5·7
Nov. 30	−0·35	−4·0

Mean Positions of the Comparison Stars for 1873·0.

	Name of Star.	Mag.	R.A. h	m	s	N.P.D. °	′	″
a	W. B. (1) IX. 278	9	9	14	25·12	89	38	3·0
b	W. B. (1) IX. 307	8	9	15	40·72	89	53	11·8

The observation on November 28 was made under unsatisfactory circumstances. The comet was excessively faint, and only visible by glimpses. On November 30, the observation was a good one.

Note on the Position of the Small Stars near ε Lyræ.

By J. M. Wilson, Esq.

The following positions were obtained in July 1872:

	°
C D	142·0
A C	173
A B	17·1
A E	134·7
A F	180·5
A G	167·5
C E	36·6
C F	338·4
C G	1·2
E F	248·9
E G	268·5
F G	39·2

On laying these out, the diagram accompanying this Note is obtained. The position of G is not accurately obtained from them, as the lines do not truly intersect in one point. The greatest error possible in its position is 2″. The faint stars would not bear any illumination of the field, and the measures were taken with a parallel-wire micrometer.

Temple Observatory, Rugby,
1874, *January* 15.

The diagram forwarded by Mr. Wilson gives the relative positions of the stars very nearly the same as in that sketched by Mr. Prince, inserted on p. 86 in the December number of the *Monthly Notices*.—[Editor.]

Le Verrier's Tables of Jupiter.

In the *Comptes Rendus* for January 12, 1874, M. Le Verrier announces that he has completed his new Tables of *Jupiter* as a supplement to his researches on the theory of the motion of that planet.

"The theory on which the Tables are founded has been developed in four papers laid successively before the Academy at the meetings of May 20, August 26, November 11, 1872, and of March 17, 1873.

"The observations with which it has been compared are those made at Greenwich between 1750 and 1830, and between 1836 and 1869, and at Paris between 1837 and 1867.

"The theory is in complete accordance with the observations.

"Hence there exists no influence other than those already known to us, which appears to have any sensible effect on the motion of *Jupiter*. The influence of the minor planets is quite insensible."

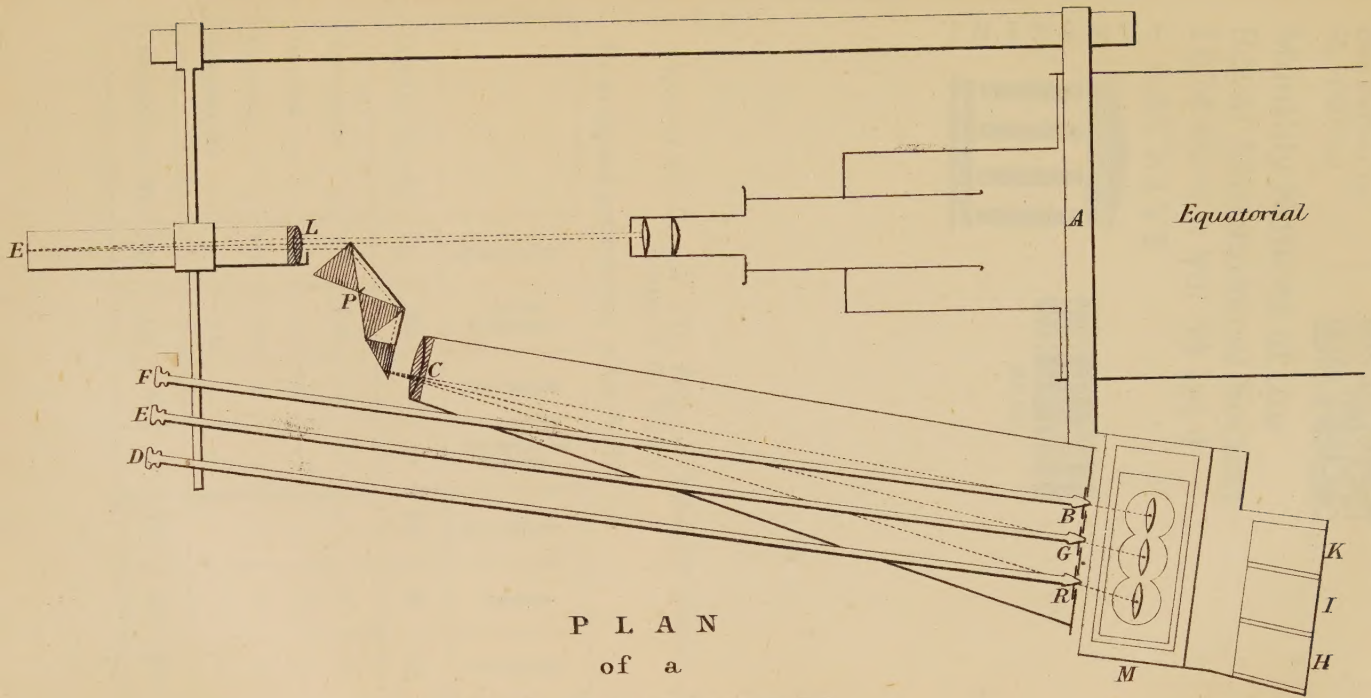
New Elements of Minor Planet (110) *Lydia.*

The following new elements of this planet have been calculated by Dr. H. Oppenheim :—

Epoch 1874, February 9·5, Berlin M. T.

	°	′	″	
L	136	18	48·7	
M	164	33	24·9	
π	331	45	23·8	Mean Equinox, 1870·0.
Ω	57	8	23·7	
i	5	59	16·2	
ϕ	4	32	46·3	
μ			781·84272	
log. a			0·437925	

With these elements Dr. Oppenheim has computed ephemerides on two hypotheses for the use of observers who may be inclined to search for the planet with an equatoreal. The computed magnitude is 11·5.



PLAN
of a
NEW PHOTOMETER.

Spottiswoode & Co. Lith, London.